Optimal Behavior of an Investor in Option Market

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Abstract - The paper deals with the problem of optimal behavior of an investor in the option market with own opinion on market properties. We tell the difference between investor's and market probability distribution functions of future prices of underlier. In this case, the investor might gain in the average income. If the investor, however, is guided by the presently popular Value-at-Risk criterion in the traditional form, the results may on the full market prove absurd. The point is that any investor commonly chooses a critical level as parameter of VaR-criterion that is relatively low, and so the investor receives low income with the probability equal to 1. It could hardly be investor's initial desire.

A modified continuous version of VaR-criterion is introduced that reflects market players' preferences more precisely and is free of this shortcoming. If the matter is how to use this method in practice, it is necessary to use a multistage version of VaR-criterion. An increasing function of critical incomes, possibly with some parameters, given for all critical probabilities in the segment [0,1] is considered. VaR-criterion for every point in this segment as far as possible starting from the point zero is required to be satisfied.

To solve this problem, the Neuman-Pearson statistic criterion with the likelihood ratio formed by market and investor's probability densities is used. A clear procedure that determines whether the problem may be solved completely or partly is given. An example of two-sided exponential probability distribution with different parameters for the investor and the market and with the power function of critical incomes demonstrates peculiarities of constructions proposed.

An approximation technique is considered to adapt the method to discrete-in-strikes option market. The exposition is concluded with a generalization of the method to multi-period option markets. The topic of the paper is partly reflected in [1].

I. THEORETICAL ONE-PERIOD OPTION MARKET

At first, the theoretical one-period option market that covers only two moments of time is considered. Transaction costs are ignored. The investor invests money at initial time and receives income at final time. In the market, standard European call options $\boldsymbol{C}(E)$ and put options $\boldsymbol{P}(E)$ of any strike $E\in R$ (set of all real numbers) trade. $\boldsymbol{U}$ denotes the risk-free asset of unit size, and $\boldsymbol{S}$ denotes the underlying stock.

Using these instruments as building blocks allows replicating various portfolios including continuous combination. So, almost any instrument $\boldsymbol{G}$ with the payoff $g(x)$ of rather arbitrary type may be replicated. The essence of the problem is that *to every linear combination of payoffs including continuous one, there corresponds the same combination of instruments.* For example, if $g(x)$ at $x \to \pm\infty$ is restricted, the presentation

$$\boldsymbol{G} = g(\nu)\boldsymbol{U} - g'(\nu-0)\boldsymbol{P}(\nu) + g'(\nu+0)\boldsymbol{C}(\nu) + \int_{-\infty}^{\nu} \boldsymbol{P}(x)dg'(x) + \int_{\nu}^{\infty} \boldsymbol{C}(x)dg'(x) \qquad (1)$$

is valid for arbitrary $\nu\in R$. On the other hand, if the derivative $g'(x)$ turns to infinity at $x=\nu$, then the following presentation is more suitable:

$$\boldsymbol{G} = g(\nu)\cdot \boldsymbol{1} - g'(-\infty)\boldsymbol{P}(\nu) + g'(+\infty)\boldsymbol{C}(\nu) + \int_{-\infty}^{\nu} (\boldsymbol{P}(x) - \boldsymbol{P}(\nu))dg'(x) + \int_{\nu}^{\infty} (\boldsymbol{C}(x) - \boldsymbol{C}(\nu))dg'(x). \qquad (2)$$

Most universal presentation (but not quite convenient one in practice) is given by formula

$$\boldsymbol{G} = \int_{-\infty}^{\infty} g(x)\boldsymbol{C}''(x)dx = \int_{-\infty}^{\infty} g(x)\boldsymbol{P}''(x)dx\,, \qquad (3)$$

where $\boldsymbol{C}''(E)$ and $\boldsymbol{P}''(E)$ are instruments of common payoff equal to δ-function relative to E that may be called "second derivatives in strike" (in the sense of differential calculus) of call and put, respectively. These instruments coincide, and so either is designated in a different way by $\boldsymbol{D}(E)$.

The stock price is equal to a given constant μ_0 at initial time and is a random variable S with the probability density $f(x)$ and the mean μ at final time. In theoretical construction, it is convenient to assume that S may take on also negative values (with low probability).

In the *risk-neutral market* all instruments have to bring in the same average relative return r (or yield $r - 1$), and so $\mu = r\mu_0$. In such a market, the price $C(E)$ of the call $\boldsymbol{C}(E)$ of strike E at initial time is determined by

$$C(E) = (1/r)\int_{E}^{\infty} (x-E)\, f(x)dx\,. \qquad (4)$$

Similar relationship is valid for put:

$$P(E) = (1/r)\int_{-\infty}^{E} (E-x)\, f(x)dx\,. \qquad (5)$$

As a consequence,

$$C''(E) = P''(E) = f(E)/r\,. \qquad (6)$$

Clearly, in such a market, the probability density $f(x)$ may be reconstructed by employing option prices.

But the *real market* is quite another matter. For the real market, (4), (5), and (6) are vacuous. Moreover, the probability distribution of future stock prices is unknown. Only option prices are available. Nevertheless, the formula (6) may be applied to determine its right part. Note that the principle of arbitrage inadmissibility calls for a certain compatibility of call and put prices.

Let us introduce normalizing factor r by the equality

$$\int_R C''(x)dx = \int_R P''(x)dx = 1/r \ .$$

Then the function

$$f_m(x) = rC''(x) = rP''(x)$$

is nonnegative, and its integral over R is equal to 1. So, it may be viewed as a probability density. This density is reasonable to denote *the implied probability density of the stock price*. Therefore, the parameter r is *the implied relative return* and r–1 is *the implied yield*. At the same time, it is clear that the price of $\boldsymbol{U}$ is equal to 1/r. Hence, r and r–1 coincide with *the risk-free relative return* and *the risk-free yield*, respectively.

II. OPTIMAL PORTFOLIO OF AN INVESTOR IN THE ONE-PERIOD OPTION MARKET

We assume that the investor has own opinion on market probability distribution of the future stock prices. We let $f_t(x)$ denote the probability density of this distribution. Also, the investor is supposed to know the implied probability density function $f_m(x)$. Let us consider investor's objectives and how to achieve them.

A. Unconditional maximizing average income of an investor

The primary problem of an investor is seemed to be unconditional maximizing the investment return. In essence, such problem may be of interest only for a risk-neutral investor. We suppose that the investor has available an amount A to invest in option market. In accordance with (3) the market price $|\boldsymbol{G}|$ of an instrument $\boldsymbol{G}$ with payoff $g(x)$ is equal to

$$|\boldsymbol{G}| = (1/r)\int_{-\infty}^{+\infty} g(x)\, f_m(x)dx \ .$$

If the investor is assumed to use only nonnegative payoffs, then the maximum return is achieved by instruments in class $\boldsymbol{D}(E)$, $E \in R$. Let us introduce the likelihood function applied in statistics

$$L(x) = f_m(x)/ f_t(x) \ .$$

Then the average income of an investor is maximized at the strike E' that achieves the minimum of the function $L(x)$. This maximum income is equal to $A/L(E')$.

If the investor is allowed to take short positions in instruments $\boldsymbol{D}(E)$, he can, theoretically, arbitrarily enhance his own average income. Actually, if the investor takes the long position in $\boldsymbol{D}(E')$ of size $A + A_s$ (dollar value) and the short position in $\boldsymbol{D}(E'')$ of size A_s, where E'' is the strike that achieves the maximum of the function $L(x)$, then his total average income is $(A+A_s)/L(E') - A_s/L(E'')$. This income is readily seen to become infinitely large as A_s increases.

B. Conditional maximizing average income of an investor (VaR-criterion)

The flaw of the above solution is obvious - it is degenerate. The investor receives zero income with probability 1 and infinite income with probability 0. Strictly speaking, this property in its pure form is only of theoretical interest, but to a certain extent, it can show up in the real market too. To manage this problem, we'll try to apply the popular criterion Value-at-Risk. Its standard form assumes maximizing average income of an investor too, but now under condition

$$P_t\{B \geq B_{cr}\} \geq 1 - \varepsilon^\circ \ , \qquad (7)$$

where a critical income B_{cr} and a fixed probability $\varepsilon^\circ \in [0,1]$ are chosen by the investor, ε° being typically small, as well as the critical level B_{cr} being relatively low.

The solution of this problem is based on Neuman-Pearson criterion and uses again the function $L(x)$ (see, for example, [2]). A one-parameter embedded system of sets $\{Z(c),\, c>0\}$ is being constructed by the rule

$$x \in Z(c) \quad \Leftrightarrow \quad L(x) \geq c \ .$$

Further, the set $X^\circ \in \{Z(c)\}$ such that $\varepsilon^\circ = P_t\{X^\circ\}$ is determined. It turns out that the implied probability $\gamma = P_m\{X^\circ\}$ is maximized among all sets X such that $P_t\{X\} = \varepsilon^\circ$. Hence the market price of the instrument that may be called "the indicator of the complement of X°" equal to 1–γ is minimized. Using B_{cr} units of this instrument ensures fulfilling (7) subject to the constraint $B_{cr}(1-\gamma) \leq A$. The residual $A - B_{cr}(1-\gamma)$ is aimed at maximizing the average income, i.e. buying the instrument $\boldsymbol{D}(E')$. Again, taking short position in $\boldsymbol{D}(E'')$ accompanied by adequate increasing the size of the long position in $\boldsymbol{D}(E')$ can make investor's total average income arbitrarily large.

C. Continuous version of VaR-criterion

This approach still may not be satisfactory for the investor. As a matter of fact, his random income does not exceed B_{cr} with probability 1. Besides, the increase of the average income is again provided by degenerate component of total income. Taking into account that, typically, the critical level B_{cr} is relatively low, we conclude that this result may hardly be the investor's initial desire.

This problem seems to be managed by considering a multistage version of VaR-criterion or a continuous one in theoretical market. We display here only continuous VaR-criterion, because the transition from it to multistage VaR-criterion does not present substantial difficulties.

We suppose that investor's preferences are described by a non-decreasing function of critical incomes $B_{cr}(\varepsilon)$, $\varepsilon\in[0,1]$. This function may take on negative values too and be unlimited at both ends of the interval. The maximum objective of the investor is to satisfy the condition

$$P_t\{B \geq B_{cr}(\varepsilon)\} \geq 1-\varepsilon \qquad (8)$$

for all $\varepsilon\in[0,1]$. Again, the residual is aimed at maximizing the average income, i.e. acquiring $\boldsymbol{D}(E')$. Taking a short position in $\boldsymbol{D}(E'')$ is possible too. If, on the contrary, the full solution of this problem is impossible, the priority is assigned to less values of ε.

This problem is being also solved by means of Neuman-Pearson procedure. Now, for *each* $\varepsilon\in[0,1]$, the set $X(\varepsilon) \in \{Z(c)\}$ such that $\varepsilon = P_t\{X(\varepsilon)\}$ is determined. As before, it turns out that the implied probability $\gamma(\varepsilon) = P_m\{X(\varepsilon)\}$ for each $\varepsilon^\circ\in[0,1]$ is maximized and hence the market price of the instrument "indicator of the complement of $X(\varepsilon)$" equal to $1-\gamma(\varepsilon)$ is minimized. Besides, $\gamma(\varepsilon) > \varepsilon$ for each $\varepsilon\in(0,1)$, $\gamma(0)=0$, $\gamma(1)=1$.

Examine the process of gradual investing an amount A as parameter ε is increasing from 0 to 1. One can show that the function $A(\varepsilon)$ of investment amount needed to fulfill (8) right up to arbitrary ε is given by

$$A(\varepsilon) = \int_0^\varepsilon B_{cr}(u)\,d\gamma(u) + (1-\gamma(\varepsilon))B_{cr}(\varepsilon)\,. \qquad (9)$$

If $A(1) \leq A$, then (8) holds for all $\varepsilon\in[0,1]$, and the problem is solved completely. Afterwards, the investor takes positions in $\boldsymbol{D}(E')$ and $\boldsymbol{D}(E'')$ to enhance his average income.

If the investor does not wish to use degenerate component of the income, he may apply, for example, one-parameter family $B_{cr}(\varepsilon,b)$ of critical incomes and then determine b to fulfill equality $A(1) = A$.

III. AN ILLUSTRATIVE EXAMPLE

To illustrate the above method, we consider an example of one-parameter two-sided exponential distribution with the density $f(x) = (1/2\beta)\exp(-|x|/\beta)$ both for the investor and the market.

At first, we assume that $\beta = 1$ for the market and $\beta < 1$ for the investor. That is, the investor supposes the market to be less volatile than the option prices show it.

For convenience and simplicity, in the example under consideration, we mean by x not the random stock price but the difference between the random stock price and the average price. Moreover, we take r=1.

Investor's risk preferences are given by the function of critical incomes

$$B_{cr}(\varepsilon,b) = b\varepsilon^\lambda, \quad \lambda > 0.$$

Besides, the investor proposes to aim the entire investment amount at fulfilling the restriction (8).

Realization of Neuman-Pearson procedure results in the embedded system of sets for c>0

$$Z(c) = \left\{x \,\middle|\, |x| \geq \frac{\beta}{1-\beta}\ln\frac{c}{\beta}\right\}.$$

Determining the critical set with the probability level ε, $X(\varepsilon)$, as an element of this system with the property $P_t\{X(\varepsilon)\} = \varepsilon$, gives the relation

$$X(\varepsilon) = \{x \,|\, |x| \geq -\beta\ln\varepsilon\}.$$

We also deduce that for the example under consideration

$$\gamma(\varepsilon) = \varepsilon^\beta.$$

Further, the equality

$$\varepsilon(x) = \exp\left(-\frac{|x|}{\beta}\right)$$

gives the connection between the probability level ε and the stock price x. The relationship (9) and the condition $A(1) = A$ combine to yield by simple calculation the equality

$$b = A\frac{(\beta+\lambda)}{\beta}\,.$$

Finally, we obtain the optimal payoff

$$g(x) = B_{cr}(\varepsilon(x),b) = A((\beta+\lambda)/\beta)\exp(-\lambda|x|/\beta)\,.$$

As a first approximation to this payoff, among simple option combinations, we may consider the payoff of a butterfly – symmetrical-in-strikes combination of the long strangle and the short straddle.

Setting $\theta = \lambda/\beta$, we can derive from (1) that to this payoff there corresponds an investor's optimal portfolio presented in the form

$$\boldsymbol{G} = A(1+\theta)(\boldsymbol{U} - \theta(\boldsymbol{P}(0)+\boldsymbol{C}(0)) + $$
$$+\theta^2\left(\int_{-\infty}^0 \boldsymbol{P}(x)\exp(\theta x)dx + \int_0^\infty \boldsymbol{C}(x)\exp(-\theta x)dx\right)\Bigg).$$

Passing in this formula from instruments to instruments' prices determines the value of the instrument G.

As a consequence, we can derive that the average income is equal to $A(\beta+\lambda) / [\beta(\lambda+1)] \geq A$, and the variation is equal to $A^2 \lambda^2 (\beta+\lambda)^2 / [\beta^2(2\lambda+1)(\lambda+1)^2]$. Hence, the greater is the value of the parameter λ, the less the investor is risk-averse. In other words, as λ increases, the investor requires less and less reward in exchange for a unit of risk augmentation.

Analogously, we consider an opposite example with $\beta = 1$ for the market and $\beta > 1$ for the investor, which means that the investor supposes the market to be more volatile than the option prices show it. In this case, we could show by the same way as above that the optimal instrument's payoff is

$$g(x) = A\frac{\Gamma(\lambda+\beta+1)}{\Gamma(\lambda+1)\Gamma(\beta+1)}\left(1-\exp\left(-\frac{|x|}{\beta}\right)\right)^{\lambda}.$$

where $\Gamma(\nu)$ is the gamma function. As a first approximation to this payoff, among simple option combinations, we may this time consider the payoff of a reversed butterfly (or sandwich) – symmetrical-in-strikes combination of the long straddle and the short strangle.

The presentation of the optimal portfolio depends on the value of parameter λ. If, for instance, $\lambda<1$, we have to apply for optimal portfolio of the investor the presentation (2), which yields

$$\boldsymbol{G} = b(\lambda/\beta)^2\left(\int_{-\infty}^{0}(\boldsymbol{P}(x)-\boldsymbol{P}(0))s(-x)dx + \int_{0}^{\infty}(\boldsymbol{C}(x)-\boldsymbol{C}(0))s(x)dx\right)$$

where

$$s(x) = \left(1-\exp\left(-\frac{x}{\beta}\right)\right)^{\lambda-2}\exp\left(-\frac{x}{\beta}\right)\left(\exp\left(-\frac{x}{\beta}\right)-\frac{1}{\lambda}\right),\quad x>0.$$

IV. ADAPTATION OF CONTINUOUS VAR-CRITERION TO REAL OPTION MARKET

Unlike theoretical markets, only discrete strikes trade in real option markets. We assume that in the market only n strikes with the price step of size h trade. We also let I denote the set of all inner strikes, that is, all strikes except the lowest one and the highest one. The implied probability density can not be reconstructed now from option prices precisely but only approximately. Hence, using available option prices can determine the implied probability density of asset prices only approximately. Therefore, the technique proposed below and the "optimal" portfolio of the investor are approximate too.

Besides, in the real market, the deep-in-the-money options do not trade. So, we mark out a certain middle reference strike a that is assumed to trade in the market both for calls and puts. We also assume that strikes left to a trade only for puts and strikes right to a trade only for calls. For any strike we may define an estimation of the probability density as a second-order difference of puts and/or calls.

So, if $i < a$, $i \in I$, we write down estimations of probability density $\tilde{f}(i)$ in terms of only put prices

$$\tilde{f}(i) = (P(i-1) - 2P(i) + P(i+1))/h^2 \,. \qquad (10)$$

Similar formulas may be written down for $i > a$, $i \in I$, in terms of only call prices

$$\tilde{f}(i) = (C(i-1) - 2C(i) + C(i+1))/h^2 \qquad (11)$$

and for $i = a$ in terms of both put and call prices:

$$\tilde{f}(i) = \frac{1}{h^2}(C(i+1) - C(i) - (P(i) - P(i-1) - h)). \quad (12)$$

As in preceding sections, the investor has available an own probability density function $f_t(x)$. We now form the likelihood ratio by the relation

$$\tilde{L}(i) = f_t(i)/\tilde{f}_m(i), \quad i \in I,$$

and arrange all strikes in descending order of this ratio, letting π_i be the order number of strike i.

When constructing the optimal portfolio, we use as building blocks elementary butterflies $\tilde{\boldsymbol{D}}$, which are formed by neighboring strikes and are a discrete analog of instruments $\boldsymbol{D}(E)$. For example, according to (10), if $i < a$, $i \in I$, then

$$\tilde{\boldsymbol{D}}(i) = (\boldsymbol{P}(i-1) - 2\boldsymbol{P}(i) + \boldsymbol{P}(i+1))/h^2 \,. \qquad (13)$$

Analogous formulas may be written for $i \geq a$, $i \in I$, with the help of (11) and (12).

We construct next an embedded set system $\{X_k\}$ of real strikes, where X_k corresponds to k least values of $L(i)$, and form an increasing sequence of critical incomes B_k derived from $B(\varepsilon)$ as follows. We attribute the investor's "probability" $h\tilde{f}_t(i)$ to each strike i, $i \in I$, and hence to every set X_k there corresponds the investor's "probability"

$$\varepsilon_k = \tilde{P}_t\{X_k\} = h\sum_{i=1}^{k}\tilde{f}_t(\pi_i).$$

Now we set $B_k = B(\varepsilon_k)$.

Finally, the "optimal" portfolio of the investor is given in the form

$$\boldsymbol{G} = h(B_1\tilde{\boldsymbol{D}}(\pi_1) + B_2\tilde{\boldsymbol{D}}(\pi_2) + \ldots + B_{n-2}\tilde{\boldsymbol{D}}(\pi_{n-2})).$$

When applying the formula (13) for $i < a$ together with analogous formulas for $i \geq a$, this presentation of portfolio may be rewritten, on request, directly in terms of calls and puts.

V. OPTIMAL PORTFOLIO OF AN INVESTOR IN THE MULTI-PERIOD OPTION MARKET.

The method above can be generalized to the multi-period option market that may be considered as an approximate model of a continuous-in-time market. Unfortunately, for this generalization could be carried out, it is not sufficient even if standard options of each possible expiration date trade in the market, because prices of such options does not contain full information about the joint probability density of stock prices at sequential instants of time.

To realize the generalization, it is necessary to introduce into the market so-called *path-dependent options*. In fact, a special type of these options called barrier options trades in real markets. Here, we interpret path-dependent options broadened. By such options we mean instruments whose payoffs depend on the entire trajectory of the stock price movement.

Among all path-dependent options we pick out basis instruments $\mathbf{A}(E,\alpha)$, called here A-options (alpha-options), that are specified by the vector $E = (E_1, E_2,\ldots, E_n)$ as a sequence of strikes and the vector $\alpha = (\alpha_1, \alpha_2,\ldots, \alpha_n)$ as an arbitrary sequence of –1 and +1 that defines the type of the option. Letting $x = (x_1, x_2,\ldots, x_n)$ be any time sequence of stock prices, we may define the payoff of $\mathbf{A}(E,\alpha)$ as the income of $\max(0, \alpha_1(E_1-x_1)) \ldots\cdot\max(0, \alpha_n(E_n-x_n))$ that is paid at time n. Instead of stock prices, relative returns might be examined.

If we form two mixed derivatives of this payoff with respect to each strike of the first and the second orders, then we get two instruments – "first- and second-order derivatives in strikes" of A-option. In particular, "first-order derivative" of the A-option with the vector $\alpha°$, all components of which are equal to +1, has the payoff equal to the joint distribution function of the stock prices vector x.

Besides, the payoffs of second-order derivatives of all A-options coincide and are equal to n-dimensional δ-function relative to E. Hence we derive the equation

$$\boldsymbol{G}\{g\} = \int_{R_n} g(x)\boldsymbol{D}(x)dx\,,$$

where $\boldsymbol{D}(E)$ is an instrument "n-dimensional δ-function" and integration is taken over all n-dimensional Euclid space R_n.

In the risk-neutral market, the price $\mathrm{A}(E,\alpha)$ of a n-period option $\mathbf{A}(E,\alpha)$, for example in the case of $\alpha=\alpha°$, may be given by relationship

$$\mathrm{A}\left(E, \alpha°\right) = \left(1/r\right)\int_{R_n} \prod_{i=1}^{n}\left(\max(0, E_i - x_i)\right) f(x)dx\,,$$

where r is the risk-free relative return over entire investment horizon.

On the other hand, if the probability distribution of asset prices is unknown, we use option prices themselves. The implied n-dimensional joint probability density is defined as (recall that the payoffs of the second-order derivatives of all A-options doesn't depend on α)

$$f_m(x) = f_{m,n}(x) = r_0^n\,\frac{\partial^{2n}\mathrm{A}(x;\alpha)}{\partial x_1^2\ldots\partial x_n^2}\,,$$

where

$$r_0^n = r = 1\Big/\int\frac{\partial^{2n}\mathrm{A}(x;\alpha)}{\partial x_1^2\ldots\partial x_n^2}\,dx$$

is interpreted as *the implied risk-free relative return* over the n-period interval.

So, starting from option prices, we can determine both the implied n-dimensional joint probability density $f_m(x)$ and the implied n-period risk-free relative return. This probability density allows determining also the implied joint probability densities $f_{m,k}(x)$, $x=(x_1,\ldots, x_k)$, $k<n$, by the rule:

$$f_{m,k}(x_1,\ldots,x_k) = \int_{R_{n-k}} f_{m,n}(x_1,\ldots,x_n)dx_{k+1}\ldots dx_n\,. \quad (14)$$

The only presence of n-period path-dependent options in the market is not sufficient to reconstruct the implied risk-free relative return over the first k periods for arbitrary $k<n$. To have such an opportunity, the presence of path-dependent options of all expiration dates $k<n$ in the market is required. Moreover, the presence of ordinary European options of all expiration dates $k<n$ in the market would be enough as well.

Still note that the path-dependent options of an expiration date $k<n$ engender the implied joint probability density $f'_{m,k}(x)$ too. It means that the implied joint probability densities $f_{m,n}(x)$ and $f'_{m,k}(x)$, $k<n$, must be mutually compatible. So $f_{m,k}(x)$ from (14) and $f'_{m,k}(x)$ must coincide, because, otherwise, temporal arbitrage is possible.

Now, the optimal portfolio of a n-period investor with own opinion on market properties is constructed as formerly, but this time by means of n-dimensional Neuman-Pearson procedure with the likelihood function

$$L(x) = f_{m,n}(x)\big/ f_{t,n}(x)\,.$$

VI. CONCLUSIONS

In the paper a well-known market paradigm is considered. If the investor assumes the market will be more volatile or less volatile than the option prices show that, he might use such instruments as long or short straddles, strangles or butterflies of some size and some type. However, a question comes to mind. What theory may suggest an instrument that will be the most preferable to the investor in such a case?

The investigation carried out in the paper proposes such a theory. The method developed is of constructive nature and may be put into shape of computing procedure. So any investor with own opinion on market properties obtains effective means of making deliberate decisions in the option market.

The investor is supposed to have own risk preferences too. The optimal portfolio is determined both by investor's view on market properties and by investor's preferences. The one-period option market is considered. Investor's market opinion is expressed in the form of the probability density of stock prices.

As emphasized, it is inappropriate to formulate investor's risk preferences in terms of the presently popular Value-at-Risk criterion in the traditional form, because when stating the problem rigorously the results may on the rich market prove absurd.

To manage this problem, the conception of continuous VaR-criterion is developed. According to this criterion, investor's risk preferences are given not by single critical income that corresponds to some critical probability level but by an increasing function of critical incomes given for all critical probabilities in the segment [0,1]. It means that investor's risk preferences are given, actually, by the probability distribution function of his desirable random income.

A method based on the Neuman-Pearson procedure is elaborated that enables to determine whether the investor's problem can be solved fully and, if so, whether his random income contains a degenerate component. Whatever the case, the optimal portfolio of an investor in the option market is constructed. If the investor does not wish to have a degenerate portfolio, he might formulate his risk preferences with the help of one-parameter function of critical incomes and consequently choose the parameter to eliminate such a component. The example considered demonstrates that the optimal portfolios of an investor can be very similar to well-known instruments such that butterfly or reversed butterfly but not exactly the same, which is the consequence of more detailed structure if his risk preferences.

A technique that allows applying the method to discrete-in-strikes option market is developed. Also, a generalization of the method to multi-period option markets with introducing a special class of path-dependent options is considered.

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