Аннотация

В работе излагаются особенности влияния длины инвестиционного горизонта на выбор инвестором оптимального для себя портфеля. Приводятся условия, при которых неэффективные портфели для однопериодного случая становятся эффективными с увеличением инвестиционного горизонта. Сравниваются два различных подхода к описанию предпочтений инвестора при выборе портфеля. Один из подходов основывается на кривых безразличия, а другой — на критерии "допустимых потерь".

Classes of preferences of portfolio investors for multi-period case and their asymptotic properties

Agasandian G.A.
E-mail: agasand@ccas.ru

The methods of the definition of preferences of portfolio investors for the multi-period investment horizon are considered and the dependence of the investor behavior on the horizon length is studied. It is assumed that the capitalization share of each portfolio security doesn't vary in time. Hence the portfolio restructuring on each step of the investment process is necessary to allocate the whole portfolio value between the component securities in a proportion chosen by investor. It is supposed that the restructuring transaction costs are equal to zero.

In portfolio theory, different approaches are used. In this paper, three of them are considered. The first one involves the definition of the effective portfolio set, the second one involves the concept of the indifference curves and the third one involves the drawdown criteria. To illustrate all these concepts, the example of the simple portfolio is applied, which consists only of two securities #1 and #2 with \( m_1 > m_2, s_1 > s_2 \), where \( m \) – the expected relative return (1 plus the period yield) of a security and \( s^2 \) – its variation. The relative returns of each security for different periods are supposed to be stochastically mutual independent and to have equal lognormal probability distributions. It is worth to note that the supposition about lognormality of distributions is to some extent excessive when solving the optimal portfolio problem for large investment horizon. It is because of validity of central limit theorem of probability theory in accordance with which under wide conditions the production of independent random values has asymptotic lognormal distribution. Strictly speaking, this consideration is of validity to less extent for the first statement below. The following properties can be formulated:

1. If \( \frac{s_2}{s_1} - \frac{m_2(m_1 - m_2)}{s_1 s_2} < \rho < \frac{s_2}{s_1} \) where \( \rho \) is a correlation coefficient between two security's relative returns, then (1) there exist non-effective one-period portfolios and (2) all these portfolios become effective when \( T \) is sufficiently large. That is, there exist portfolios of no interest for a risk-averse investor for one-period case and these portfolios are of certain interest when the investment horizon length is greater than one period.

2. The simplest method of using indifference curves in the case \( T = 1 \) involves the consideration of lines of the type \( m_p - As_p^2 = \text{const} \), where \( A \) is the investor's individual risk-aversion coefficient – the greater it is the more the investor is risk-averse. Obviously, it is illegal to use such curves in the case \( T > 1 \) if not to transform the constants \( A \) in the proper way. Note
that the following asymptotic representations for the expected value and for the variance of the averaged-in-time portfolio relative return take place when \( T \to \infty \): 
\[
\mu_p = \frac{m_p^2}{\sqrt{s_p^2 + m_p^2}} \quad \text{and} \quad 
\sigma_p^2 = \frac{1}{T} s_p^2 + m_p^2 \left[ \ln \frac{s_p^2 + m_p^2}{m_p^2} \right] 
\]
correspondingly. So it is reasonable for sufficiently large \( T \) to consider the lines \( \mu_p - AT \sigma_p^2 = \text{const} \) as indifference curves, i.e. to consider the new coefficient \( AT \) instead of the coefficient \( A \). In this case, the coefficient \( A \) becomes the universal constant for any length of the investment horizon that characterizes the individual investor's inclination to the risk.

3. In accordance with the drawdown criteria, the goal of the investor is to define the portfolio that maximizes the expected value of the averaged-in-time portfolio relative return for \( T \) periods provided that 
\[
\Pr \{ r_p(T) < r^\circ(T) \} \leq \gamma
\]
where \( \gamma \) is a fixed significance level, \( r^\circ(T) \) is the minimal level of the relative return acceptable for investor, \( r_p(T) \) is the relative return for \( T \) periods. The combination of the parameter \( \gamma \) and the quantity \( r^\circ(T) \) characterizes the extent of the investor's aversion to the risk. In the case under consideration the probability restriction boils down to the inequality 
\[
a + \frac{z b}{\sqrt{T}} \geq \frac{\ln r^\circ(T)}{T}
\]
where 
\[
a = \frac{1}{2} \ln \frac{m_p^4}{s_p^2 + m_p^2}, 
\]
\[
b = \ln \frac{s_p^2 + m_p^2}{m_p^2}
\]
and 
\[
z = z(\gamma)
\]
is \( \gamma \)-fractile of the standard normal random value. To obtain an asymptotic form of the drawdown criteria, it is necessary to assign the parameter \( Z \) and the drawdown level \( R \) and then to define \( z \) and \( r^\circ(T) \) according to the relationships 
\[
Z = \frac{z}{\sqrt{T}} \quad \text{and} \quad R = \frac{\ln r^\circ(T)}{T}, \quad \text{i.e.} \quad z = Z \sqrt{T} \quad \text{and} \quad r^\circ(T) = \exp(RT).
\]
These quantities \( Z \) and \( R \) define the investor aversion to the risk completely – the greater \( Z \) under given \( R \) or the less \( R \) under given \( Z \) the less the investor is risk-averse.